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VECTOR WIND PROFILE GUST MODEL

FINAL REPORT

(For Period April 10, 1980 -- April 9, 1981)

Prepared For

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

Under Contract NAS8-33433

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SECTION I. INTRODUCTION

This report summarizes the work under Contract NAS8-33433 during the 12-month period beginning April 10, 1980. The detailed background information concerning earlier work on this contract can be found in References 1 and 2. The objective of the study is the development of a vector wind gust model that is suitable for orbital flight test operations and trade studies. During this reporting period, emphasis has been given to verification of the hypothesis that gust component variables are gamma distributed, gust modulus is approximately Weibull distributed, and zonal and meridional gust components are bivariate gamma distributed. The body of the report is contained in four sections (II through V). Section II describes a method of testing for bivariate gamma distributed variables; in Section III, two distributions for gust modulus are described and the results of extensive hypothesis testing of one of the distributions are presented; Section IV establishes the validity of the gamma distribution for representation of gust component variables. Conclusions are presented in Section V.

SECTION II. TESTING FOR BIVARIATE GAMMA DISTRIBUTED VARIABLES

The hypothesis that absolute component gust and associated gust length are bivariate gamma distributed can be tested according to the procedure described below.

Consider variables x and y that are distributed according to the bivariate gamma distribution with scale parameter β_1 and β_2 ; dimensionless variables T_1 and T_2 are defined by

$$\begin{aligned} T_1 &= \beta_1 x \\ T_2 &= \beta_2 y \end{aligned} \tag{1}$$

The variables T_1 and T_2 can be expressed in a coordinate system that is rotated by 45° ; the transformed variables z_1 and z_2 are given by

$$\begin{aligned} z_1 &= \frac{\sqrt{2}}{2} (T_1 + T_2) = \frac{\sqrt{2}}{2} (\beta_1 x + \beta_2 y) \\ z_2 &= \frac{\sqrt{2}}{2} (T_2 - T_1) = \frac{\sqrt{2}}{2} (\beta_2 y - \beta_1 x) \end{aligned} \tag{2}$$

It can be shown that the probability, P_Δ , that bivariate gamma distributed variables z_1 and z_2 will occur within the area bounded by the lines $z_1 = z_1^*$, $z_1 = z_2$, and $z_1 = -z_2$ (illustrated in Figure 1) can be estimated from the series:

$$P_\Delta = \frac{(1-\rho)^\gamma}{\Gamma(\gamma)} \sum_{m=0}^{\infty} \frac{\rho^m}{m!} \Gamma(\gamma+m) H\left(2(\gamma+m), \frac{\sqrt{2}}{1-\rho} z_1^*\right) \tag{3}$$

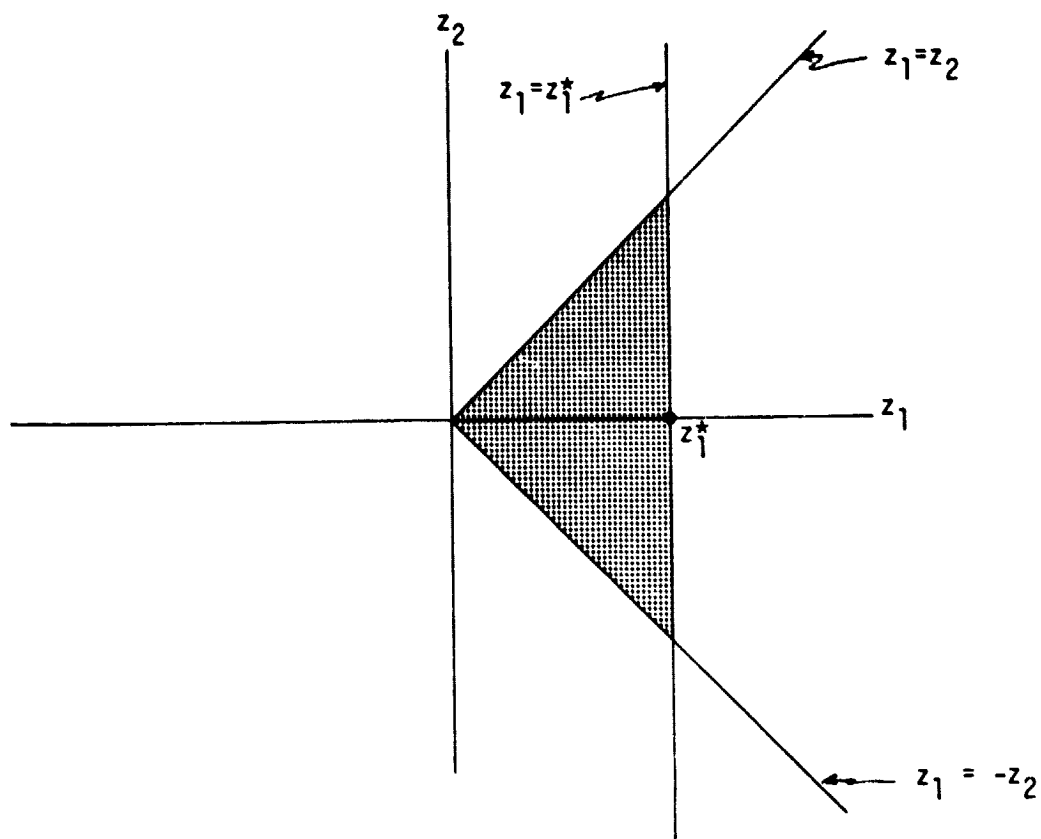


Figure 1. Area, Δ (Shaded), Which Bounds Bivariate Gamma Distributed Variables z_1 and z_2 for Which a Probability of Occurrence Can Be Calculated from Equation 3

$H(a, X)$ is the incomplete gamma function which is given by the series

$$H(a, X) = X^a e^{-X} \sum_{n=0}^{\infty} \frac{X^n}{\Gamma(a+n+1)} \quad (4)$$

where $a = 2(\gamma+m)$

$$X = \frac{\sqrt{2}}{1-\rho} z_1^*$$

ρ = correlation between variables x and y

Alternatively, P_{Δ} can be calculated by numerical integration of the equation

$$P_{\Delta} = \frac{\sqrt{2\pi} \int_0^{z_1^*} z_1^{\gamma-\frac{1}{2}} e^{\frac{-\sqrt{2}z_1}{1-\rho}} I_{\gamma-\frac{1}{2}} \left\{ \frac{\sqrt{2\rho}}{1-\rho} z_1 \right\} dz_1}{(1-\rho)^{\frac{1}{2}} (\sqrt{2\rho})^{\gamma-\frac{1}{2}} \Gamma(\gamma)} \quad (5)$$

where $I_n \left\{ \frac{\sqrt{2\rho}}{1-\rho} z_1 \right\}$ is the modified Bessel function of the first kind of order n ; for $Y = \frac{\sqrt{2\rho}}{1-\rho} z_1$, $I_n(Y)$ is calculated with the series approximation,

$$I_n(Y) = \sum_{k=0}^{\infty} \frac{Y^{n+2k}}{2^{n+2k} k! \Gamma(n+k+1)} \quad (6)$$

A computer program for calculation of P_{Δ} , using Equation (3), has been developed. A sample of the calculations of P_{Δ} as a function of ρ for $\gamma=3$ are listed in Table 1.

Table 1. $P_{\Delta}(\rho, \gamma=3)$ Calculated According to Equation 3

P CELYA - SERIES APPROXIMATION

С А М М 1 = 3 . 0

1940	.1	.2	.3	.4	.5	.6	.7	.8	.9
25	00013669	00013641	000305161	000007655	000112001	000020087	001630892	000006626	0002262874
50	000122439	000161269	000218122	000300496	000042000	000671951	001079473	001800323	0003266531
75	000106259	001200499	000166031	002198620	000979904	0004138859	000800808	0008712993	0012122510
100	000409874	000501769	000030665	001797373	001200566	0017180281	001718384	0022178395	0022209899
125	001137359	001367834	001649673	001698015	002310315	0029793530	0036265251	0043605774	0051776887
150	002862601	002854751	003957309	0039452220	0043751108	0058466478	0063939990	0072100379	0082266837
175	003572081	004061837	005983867	0062146850	0074737581	0087416003	0097837589	0108368992	0118727644
200	005559071	006217143	0094135205	009628379	0115705275	0127681469	0136431603	0149647481	0160052507
225	011355984	012451567	013611366	0146117352	016276631	0172264887	0183653356	0194702651	0204057543
250	015919511	017169926	018426645	0197180870	0209620287	0221475001	0232557615	0242788395	0252112804
275	0211018635	0224343151	0237492034	0250193277	0262161769	0273274457	0283365657	0292511900	0300818328
300	026752369	028060525	0293605171	030551378	0316471274	0326527363	0335464793	0342819243	0349086103
325	03269094	033950370	0351151954	0361701947	0371091988	0379354161	0386590198	0392931127	0398515809
350	037478123	0396678157	0417485151	0427485151	0437267845	0443765334	0451611103	0461356583	0464047242
375	0476784199	0487012288	0496968630	0471813425	0477495614	0482193664	0486999523	0489366144	0492119286
400	050624910	0513372733	0519249077	0523958317	0527677067	0530603348	0532904647	0537718946	0543615096
425	058212316	0586903146	0576601977	0573265131	0575172025	0576403409	0577217408	0577679743	0579280096
450	0614570901	0617050620	0619621890	0619179185	0619555591	0619320288	0618800426	0618008685	0617247865
475	068707070	0634333147	0682023356	0670266916	0667744571	0659220034	0657520298	0655774477	0654004201
500	0707290106	0757383106	0702694887	0701260829	0696464474	0695984162	0693325404	0689777736	0684153610
525	0747347417	0744100653	074553463	073693326	0737249516	0729693027	0726331970	0722962142	0719706416
550	0782172092	0778464164	0773764975	0769145951	0764677399	0760394178	0756307907	0752417475	0747715110
575	0814770535	0809490171	0804622945	0799115905	0793031104	0788215362	0783059786	0779484925	0775284643
600	0842555657	0836500779	0829858354	0823999026	0814488121	0813303657	0804421969	0803817041	0799463555
625	086772245	0859762599	0853176154	084709242	084137386	083528958	0829776269	0825959936	0821437053
650	0887729567	088457260	0879600890	0873169987	0864482039	085572554	0847799593	0840526382	0831320775
675	0901759590	0894442499	0891622562	0885311343	087842664	0873920493	0868747912	0863870710	0859236230
700	0921178930	0913948506	0907263016	0905276234	0895276234	0889461751	0879948316	0875306573	
725	0934775535	0924553106	0912716525	0903726571	0892738697	0882028493	0871284132	0861340806	
750	0946263134	0936622373	0926810930	0916535077	0904155031	0892703150	0880500014	0869434971	
775	0959216372	0947668734	0936866183	0925158794	0913964759	0901739614	0889437039	0877126412	
800	0974698171	096265505	0952444934	0940905506	0927676202	0915282758	0903039914	0890396945	
825	0989252085	0976500170	0963883001	09507913059	0936719086	0922622648	0909475005	0896304223	
850	0998214654	0984705328	0970531264	0956999215	094222271	0928686607	0916171302	0903466902	
875	1006132166	0992620926	0979238136	0965727302	0952768465	0939736914	0926749432	0913941178	
900	1014073705	0998566913	0982666913	096927875	0956761632	0943663760	0931229337	0918554966	
925	1020661134	0994567620	0983561999	0968748859	095011638	0937622584	0925332637	0912757370	
950	1027923141	0996973140	0985679146	0969778469	0953640033	0940717714	0928127330	09150063710	
975	1034553339	0998485757	0986575259	0969615050	0954613767	0940758617	0925153333	0910286808	
1000	1040634400	0997670809	0981566708	0964220724	0949542122	0935827563	0921950487	0907463813	
1025	104612319	0996222704	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1050	105104663	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1075	1056043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1100	1060043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1125	1064043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1150	1068043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1175	1072043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1200	1076043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1225	1080043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1250	1084043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1275	1088043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1300	1092043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1325	1096043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1350	1100043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1375	1104043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1400	1108043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1425	1112043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1450	1116043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1475	1120043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	
1500	1124043794	099446741	098044610	0964045917	0948743781	0934408120	092196860	0907345779	

Standard results for checking the computer programs are obtained from the closed form solution for the case $\gamma=1$ which can be expressed in the form

$$P_{\Delta}(\gamma=1, \rho) = 1 - \frac{e^{-\frac{\gamma}{\sqrt{\rho}}}}{2} [e^{\gamma}(\frac{1}{\sqrt{\rho}} + 1) - e^{-\gamma}(\frac{1}{\sqrt{\rho}} - 1)] \quad (7)$$

Values for P_{Δ} are listed in Table 2 for selected values of z_1^* and ρ .

Table 2. $P_{\Delta}(\gamma=1, \rho)$ Calculated from Equation (7)

z_1^*	ρ				
	.1	.25	.50	.75	.875
1	.425980	.445255	.474470	.495090	.501805
2	.774586	.774144	.769772	.763367	.760084
3	.919308	.911445	.899446	.889099	.884464
4	.971975	.965471	.956084	.948025	.944361
5	.990370	.986548	.980820	.975641	.973206
6	.996704	.994760	.991624	.988584	.987097
7	.998874	.997959	.996342	.994650	.993786
8	.999615	.999205	.998402	.997493	.997008
9	.999869	.999690	.999302	.998825	.998559
10	.999955	.999879	.999695	.999449	.999306

Another useful special case is for $\rho=0$ and 2γ equal to an integer.

$$P_{\Delta}(\rho=0, 2\gamma = \text{an integer})$$

$$= 1 - e^{\sqrt{2} z_1^*} \sum_{k=0}^{2\gamma-1} \frac{2^{k/2} (z_1^*)^k}{k!} \quad (8)$$

The variation of P_{Δ} as a function of correlation coefficient, ρ , (for $\gamma=2$) and as a function of shape parameter, γ , (for $\rho=.5$) is illustrated in Figures 2 and 3, respectively.

A comparison of observed and expected P_{Δ} is illustrated in Figure 4; the line drawn at an angle of 45° to the abscissa represents perfect agreement between observed and expected values. Deviations of the plotted points from the line represent differences between the observed and expected values. The data plotted in Figure 4 show a consistent pattern at 10 and 12 km; for $P_{\Delta} < .3$, the observed is larger than the expected; for intermediate values ($.3 < P_{\Delta} < .8$), the expected is larger than the observed. These results are the basis for initiating a more detailed analysis of the validity of the gamma distribution hypothesis for the marginal distributions (component gust and associated gust length). The results of this analysis are described in the next section.

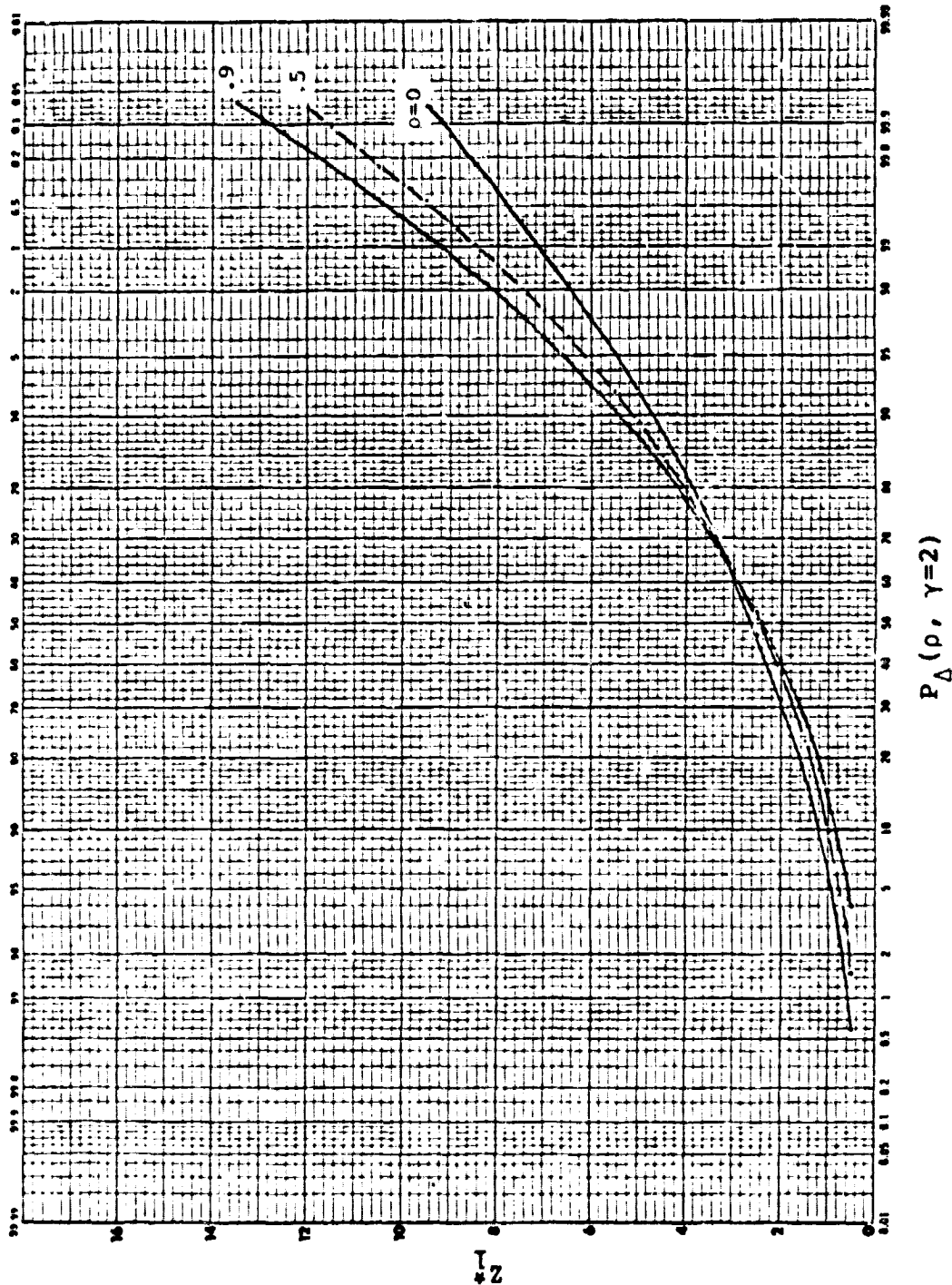


Figure 2. Series Approximation of P_{Δ} as a Function of z_1^* and ρ for $\gamma=2$

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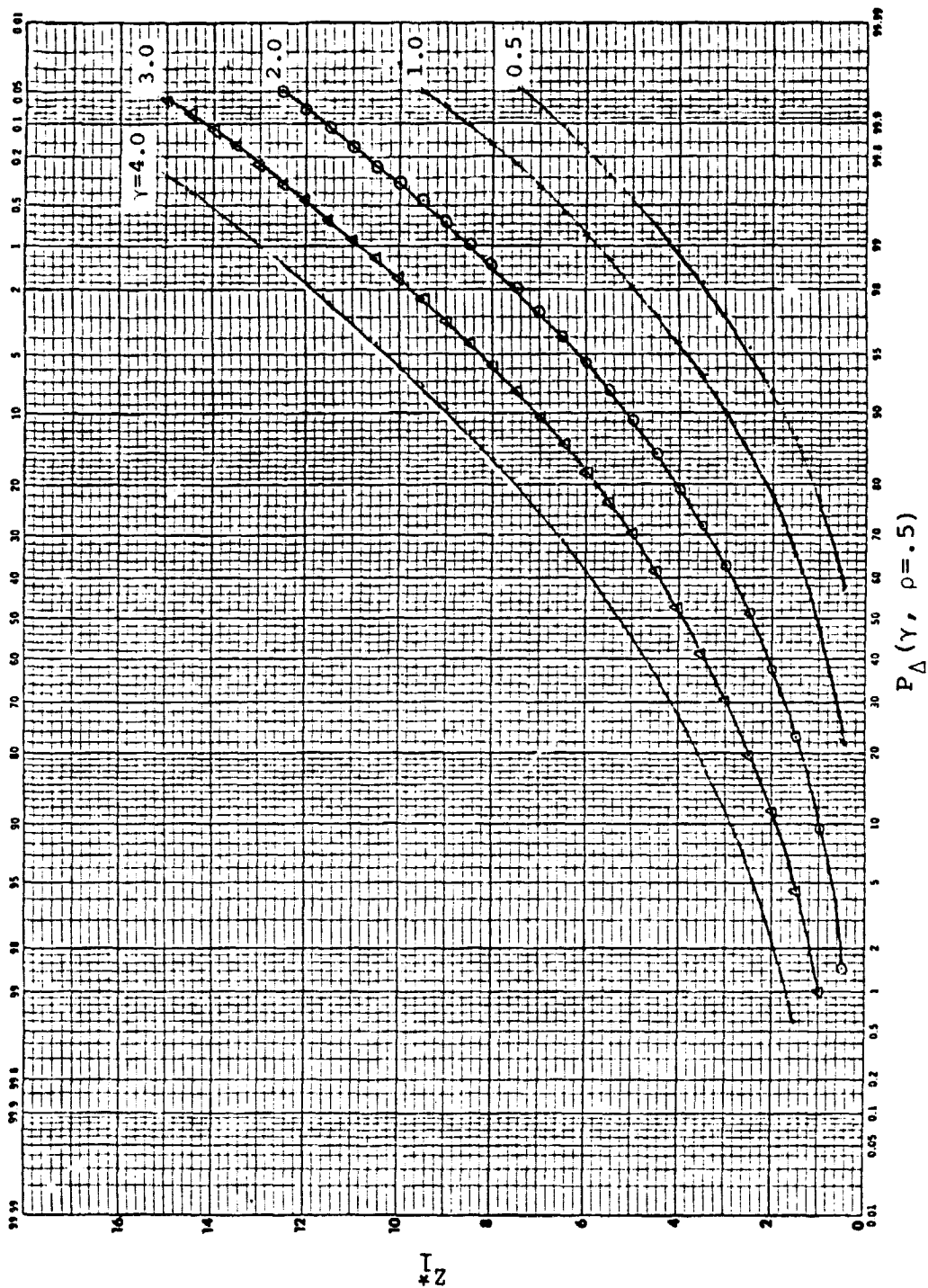


Figure 3. Series Approximation of P_Δ as a Function of z_1^* and γ for $\rho=0.5$

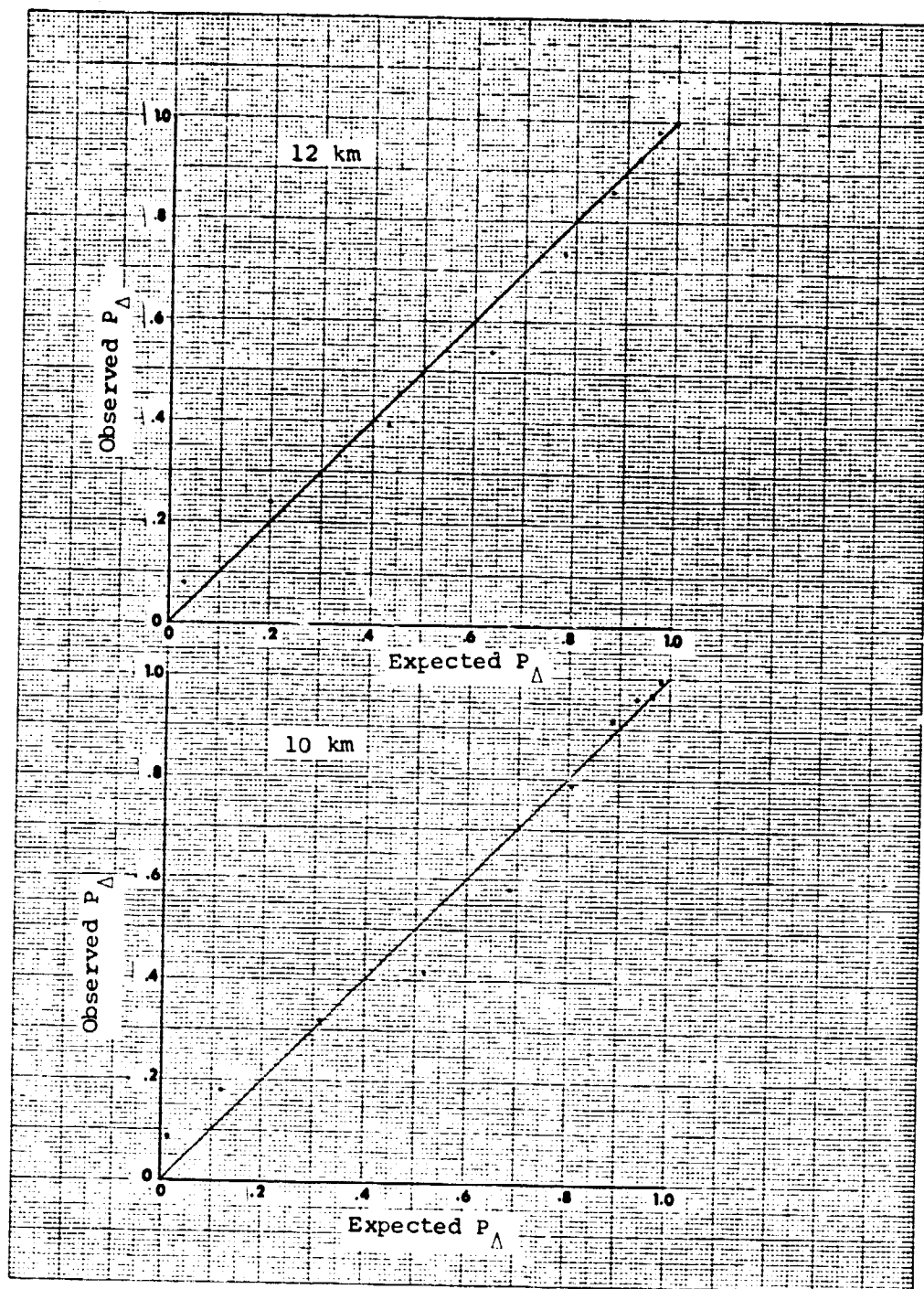


Figure 4. Observed and Expected P_{Δ} at 10 and 12 km
 Calculated from u Component Gust and Gust
 Length Data ($\lambda_c = 2470$ m) During February
 at Cape Kennedy

SECTION III. THE DISTRIBUTION OF GUST MODULUS

Given that the absolute gust components are uncorrelated bivariate gamma distributed, it follows that the probability distribution of the gust modulus, R , can be calculated with the double series approximation,

$$P_r\{R \leq R^*\} = G(R^*) = \frac{R^{*\gamma_1+\gamma_2} \beta_1^{\gamma_1} \beta_2^{\gamma_2}}{2\Gamma(\gamma_1)\Gamma(\gamma_2)} \sum_{i=0}^{\infty} \frac{(-R^*\beta_2)^i}{(\gamma_1+\gamma_2+i)\Gamma\left(\frac{\gamma_1+\gamma_2+i}{2}\right)} \cdot \sum_{n=0}^i \frac{\beta_1^n \beta_2^{-n} \Gamma\left(\frac{\gamma_1+n}{2}\right) \Gamma\left(\frac{\gamma_2+i-n}{2}\right)}{n!(i-n)!} \quad (9)$$

where β_1 and β_2 are the scale parameters and γ_1 and γ_2 are the shape parameters of the u and v component gust distributions, respectively.

Smith* has shown that the above expression is approximately equivalent to

$$G(R^*) = \frac{H(\gamma_1 + \gamma_2, AR^*)}{\Gamma(\gamma_1 + \gamma_2)} \quad (10)$$

where $H(\gamma_1 + \gamma_2, AR^*)$ is the incomplete gamma function which can be calculated accurately with the series approximation given in Section II.

*Personal communication.

$$A = \left[\frac{\Gamma(\frac{\gamma_1}{2}) \Gamma(\frac{\gamma_2}{2}) \Gamma(\gamma_1 + \gamma_2) \beta_1^{\gamma_1} \beta_2^{\gamma_2}}{2 \Gamma(\gamma_1) \Gamma(\gamma_2) \Gamma(\frac{\gamma_1 + \gamma_2}{2})} \right]^{\frac{1}{\gamma_1 + \gamma_2}} \quad (11)$$

Preliminary tests have indicated that reasonably accurate estimates of the probability distribution can be obtained from equation 10. However, it would be advantageous to determine if an alternative expression can be found which would not require as much computation. The Weibull distribution, widely used in wind energy studies (Reference 3) was chosen to represent gust modulus because of its relative mathematical simplicity and the availability of data for parameter estimation. The cumulative probability function for the Weibull distribution of gust modulus is

$$G(R^*) = 1 - \text{EXP} \left[- \left| \frac{R^*}{c} \right|^k \right] \quad (12)$$

The parameters k and c are calculated according to the approximation given by Justus (Ref. 3)

$$k = \left(\frac{\sigma_R}{\bar{R}} \right)^{-1.086} \quad (13)$$

$$c = \frac{\bar{R}}{\Gamma(1 + 1/k)} \quad (14)$$

It is noted that equation 13 implies the relation,

$$\left(\frac{\sigma_R}{\bar{R}} \right)^2 = k^{-1.84162} \quad (15)$$

whereas the exact relation for a Weibull distribution is given by

$$\left(\frac{\sigma_R}{\bar{R}}\right)^2 = \left[\frac{\Gamma(1 + 2/k)}{\Gamma^2(1 + 1/k)} \right] - 1 \quad (16)$$

The accuracy of the approximation has been evaluated for values of k from 0.5 to 10 by calculating the ratio, P , of the right side of equation (16) to the right side of equation (15). Perfect agreement is indicated when $P=1$. As illustrated in Figure 5, for $k > 1$, P is within a few percent of unity; for $k < 1$, P approaches ∞ as k approaches 0. Therefore, it is concluded that the approximation given by equation (13) is accurate for $k > 1$. The calculated values of k for gust modulus are between 2 and 3 which is within the range of acceptable accuracy of equation (13).

A comparison of the Weibull, The probability distribution associated with the modulus of a bivariate normal, and the observed probability distribution is illustrated in Figure 6. It is indicated that there is little difference between the theoretical distributions for percentiles between 20 and 98; for percentiles outside that range, the distributions diverge; for this case, the observed distribution fits the Weibull slightly better than the bivariate gamma modulus distribution.

The hypothesis that gust modulus at a reference altitude is drawn from a Weibull distributed population was tested for 69 cases. The results summarized in Table 3 indicate that the hypothesis is accepted at the .05 level of significance in a large majority (65/69) of the cases.

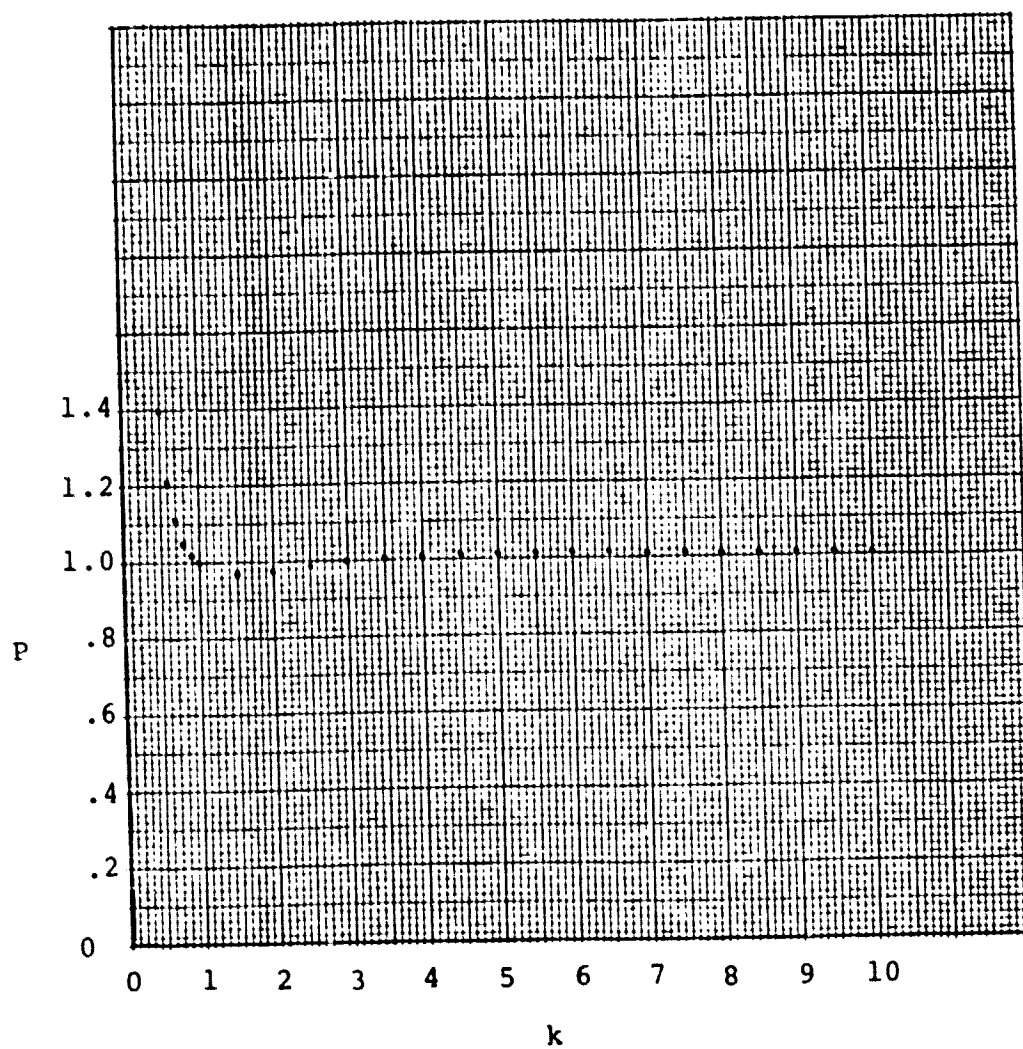


Figure 5. Ratio P as a Function of Shape Parameter, k

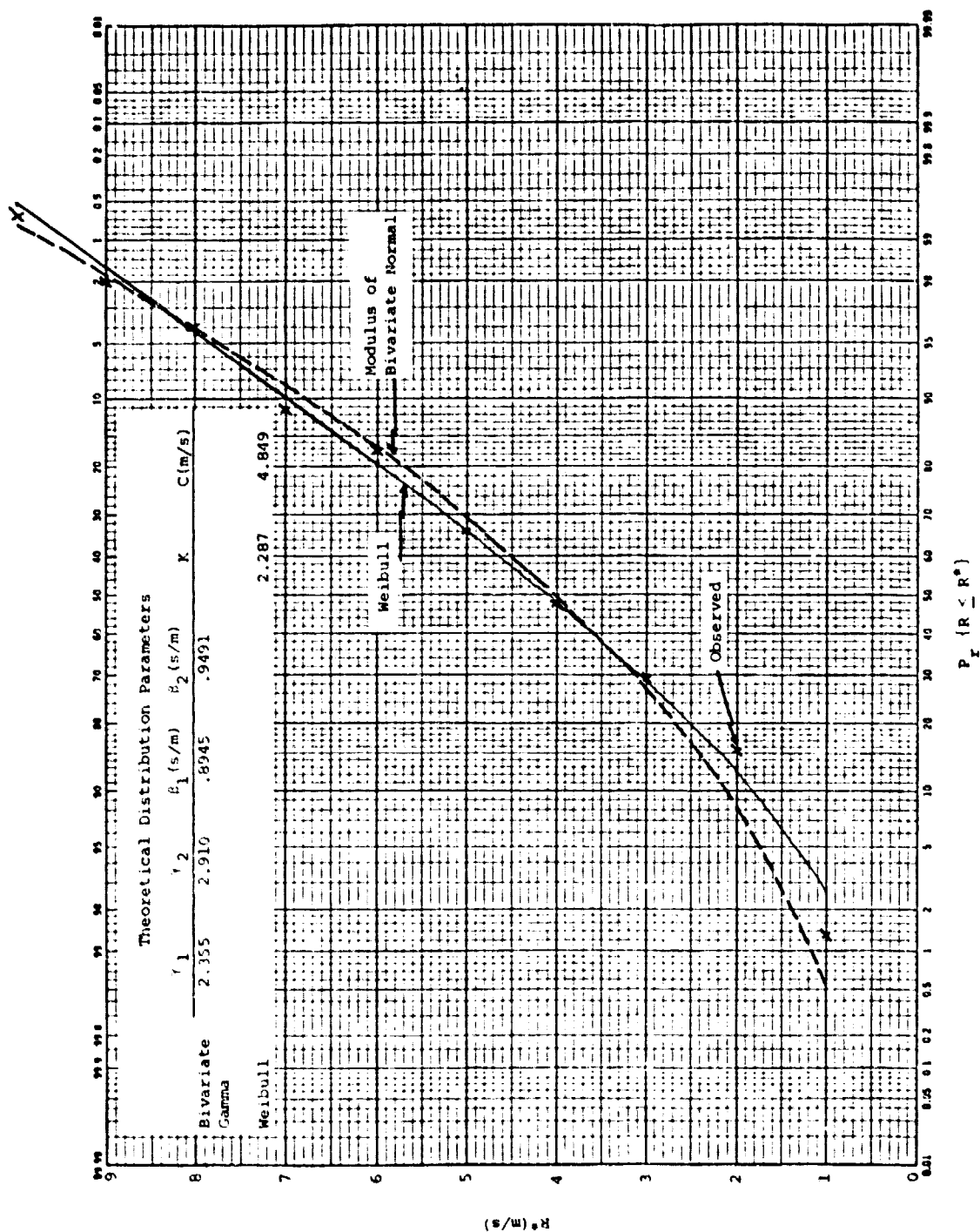


Figure 6. Observed and Theoretical Distribution of Gust Modulus at 12 km During February at Cape Kennedy for $\lambda_c = 2,470$ m

Table 3. Summary of Results of Testing the Hypothesis* That Gust Modulus at a Reference Altitude (4, 6 ... 14 km) Is Drawn From a Weibull Distributed Population

Month	λ_c (m)	Number of Cases		
		Hypothesis Accepted	Hypothesis Rejected	Insufficient Data
Feb	420	6	0	0
	997	5	1	0
	2470	5	1	0
	6000	5	0	1
	Total	21	2	1
Apr	420	5	1	0
	997	6	0	0
	2470	6	0	0
	6000	4	1	1
	Total	21	2	1
Jul	420	6	0	0
	997	6	0	0
	2470	6	0	0
	6000	5	0	1
	Total	23	0	1
Grand Total		65	4	3

*For the .05 level of significance for a χ^2 variate with m degrees of freedom, $m = r-1-b$, where r = number of class intervals, b = number of parameters of the Weibull distribution = 2.

SECTION IV. DISTRIBUTION OF GUST COMPONENT VARIABLES

Four variables associated with gusts at a reference height, H_0 , have been studied to establish the validity of the hypothesis that they are samples from gamma distributed populations. The four variables are illustrated in Figure 7. The variable u_1 is the largest u component excursion with sign equal to the sign of u at H_0 ; u_2 is the largest u component excursion of sign opposite u_1 found by scanning upward after the second zero crossing associated with u_1 . The vertical distance between u_1 and u_2 is defined as L Range; the sum of the absolute values of u_1 and u_2 is defined as u Range. The variables u Range and L Range represent wind shear and wind shear altitude interval associated with gusts in the vicinity of H_0 . Each of the four variables defined above have been calculated at six reference altitudes from a sample (150/month) of February, April, and July Jimsphere wind profile data from Cape Kennedy. These data sets were tested to establish the validity of the hypothesis that each variable is drawn from a gamma distributed population. Acceptance or rejection of the hypothesis is at the .05 level of significance for a χ^2 variate defined by

$$\chi^2 = \sum_{i=1}^N \frac{(O_i - E_i)^2}{E_i} \quad (17)$$

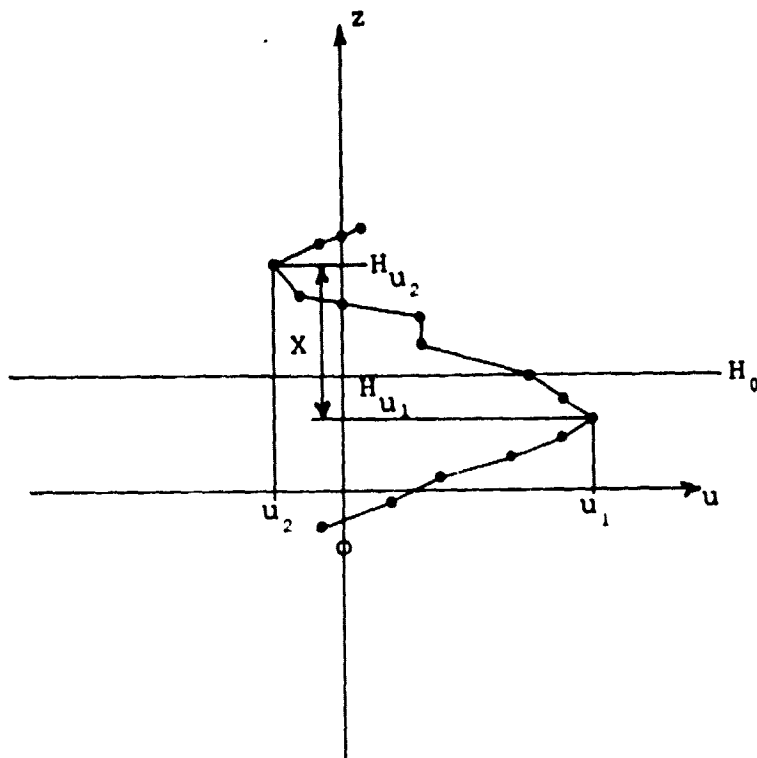
O_i = Observed frequency in the i th class interval

E_i = Expected frequency in the i th class interval
(of the theoretical-gamma distribution)

The results of the hypothesis testing are described below.

A. ABSOLUTE GUST COMPONENT AND ASSOCIATED GUST LENGTH

Gust, defined as the maximum excursion between successive zero crossings in the vicinity of a reference altitude, and associated gust length, defined as the distance between zero crossings, are each hypothesized to be drawn from a gamma distributed population. The hypothesis is accepted at the



$$u \text{ Range} = |u_1| + |u_2|$$

$$L \text{ Range} = X = H_{u_2} - H_{u_1}$$

Figure 7. Schematic Definition of u Range and L Range

.05 level of significance, in a large majority of cases, for gust component magnitude ($|u'|$); specifically, the accept/reject ratio is 47/22 and 46/23 for u and v component magnitudes, respectively. As indicated in Table 4, the ratio is significantly smaller for gust length (L_u and L_v) with rejections exceeding acceptances (for method I). The large number of rejections is attributed to large differences between observed and expected frequency of occurrence in the first few class intervals; the observed frequencies are always much larger than the expected frequencies. Small gust magnitudes are associated with small gust lengths that are observed as a consequence of the definition of gust used in this study. These small gust lengths are not measurable with the Jimsphere system; therefore, they are not considered to be valid data for hypothesis testing. By neglecting these data, we obtain the results summarized under II in Table 4 which indicate acceptance in a much larger proportion of the cases.

B. U RANGE AND L RANGE

A summary of results of testing the hypothesis that the variables, U range and L range, are drawn from gamma distributed populations is given in Table 5. It is indicated that the hypothesis for U range is accepted at the .05 level of significance in 66 of the 72 cases. Acceptance is not a function of altitude except in July when the number of samples accepted at 14 km was less than at the other altitudes. Acceptance was not related to filter choice with only slight exceptions (for $\lambda_c = 2470$ during July and $\lambda_c = 6000$ m during February one-third of the samples were rejected). Based on these results, it is concluded that U range is gamma distributed.

The results for L range summarized in the lower half of Table 5 indicate acceptance of the hypothesis (46 of the 72 cases) with not as strong a tendency as that indicated above for U range. Acceptance is an irregular function of altitude which is a minimum at 12 km where 50 percent are accepted to a maximum at 8 km where 75 percent are accepted. Acceptance is greater in July (75 percent) than in either April or February (58 percent for both months). Acceptance is weak or non-existent for $\lambda_c = 420$ m and is strong for λ_c large (2470 and 6000 m).

Table 4. Summary of Results of Testing the Hypothesis⁽¹⁾ that u and v Component Absolute Gust and Gust Length are Drawn from Gamma Distributed Populations

A/R⁽²⁾

|u'|

Filter $\lambda_c (m)$	Method							
	I				II			
	Month				Month			
	2	4	7	All Months	2	4	7	All Months
420	4/2	6/0	5/1	15/3	5/1	6/0	6/0	17/1
997	6/0	5/1	4/2	15/3	6/0	6/0	5/1	17/1
2470	5/1	1/5	2/4	8/10	6/0	4/2	5/1	15/3
6000	3/2	5/0	1/4	9/6	5/0	5/0	5/0	15/0
All Filters	18/5	17/6	12/11	47/22	22/1	21/2	21/2	64/5

Lu

420	5/1	4/2	4/2	13/5	6/0	5/1	5/1	16/2
997	1/5	1/5	4/2	6/12	2/4	1/5	6/0	9/9
2470	0/6	1/5	3/3	4/14	4/2	4/2	5/1	13/5
6000	3/2	4/1	2/3	9/6	5/0	5/0	5/0	15/0
All Filters	9/14	10/13	13/10	32/37	17/6	15/8	21/2	53/16

|v'|

Filter $\lambda_c (m)$	Method							
	I				II			
	Month				Month			
	2	4	7	All Months	2	4	7	All Months
420	6/0	4/2	6/0	16/2	6/0	4/2	6/0	16/2
997	4/2	5/1	2/4	11/7	5/1	6/0	5/1	16/2
2470	3/3	4/2	3/3	10/8	4/2	5/1	5/1	14/4
6000	3/2	4/1	2/3	9/6	5/0	4/1	3/2	12/3
All Filters	16/7	17/6	13/10	46/23	20/3	19/4	19/4	58/11

Lv

420	5/1	4/2	3/3	12/6	5/1	5/1	5/1	15/3
997	0/6	2/4	2/4	4/14	1/5	5/1	4/2	10/8
2470	1/5	3/3	3/3	7/11	3/3	4/2	5/1	12/6
6000	2/3	1/4	4/1	7/8	4/1	4/1	4/1	12/3
All Filters	8/15	10/13	12/11	30/39	13/10	18/5	18/5	49/20

(1) At the .05 level of significance for χ^2 variate with m degrees of freedom; $m = n-1-b$, where n = number of class intervals, b = number of parameters of the gamma distribution = 2.

(2) A/R is the ratio of the number of cases accepted to the number rejected.

Table 5. Summary of Results of Testing the Hypothesis That the Variables, U Range and L Range, at a Reference Altitude (4, 6, ... 14 km) are Drawn from Gamma Distributed Populations

Variable	Month	Filter λ_c (m)	Reference Altitude (km)						Summary	
			4	6	8	10	12	14	A	R
U range	2	420	A*	A	A	A	A	A	6	0
		997	A	A	A	A	A	A	6	0
		2470	A	A	A	A	A	A	6	0
		6000	R*	A	A	A	R	A	4	2
	Accept/Reject (all filters)		3/1	4/0	4/0	4/0	3/1	4/0	22 / 2	
	4	420	A	A	A	A	A	A	6	0
		997	A	A	A	A	A	A	6	0
		2470	A	A	A	A	A	A	6	0
		6000	A	A	A	A	R	A	5	1
	Accept/Reject		4/0	4/0	4/0	4/0	3/1	4/0	23 / 1	
	7	420	A	A	A	A	A	A	6	0
		997	A	A	A	A	A	R	5	1
		2470	A	A	A	R	A	R	4	2
		6000	A	A	A	A	A	A	6	0
	Accept/Reject		4/0	4/0	4/0	3/1	4/0	2/2	21 / 3	
	Accept/Reject (all months)		11/1	12/0	12/0	11/1	10/2	10/2	66 / 6	
L range	2	420	R	R	R	R	R	R	0	6
		997	A	A	R	A	R	R	3	3
		2470	A	A	A	A	A	A	6	0
		6000	R	A	A	A	A	A	5	1
	Accept/Reject		2/2	3/1	2/2	3/1	2/2	2/2	14 / 10	
	4	420	R	R	A	R	R	R	1	5
		997	A	A	R	A	R	R	3	3
		2470	A	A	R	R	A	A	4	2
		6000	A	A	A	A	A	A	6	0
	Accept/Reject		3/1	3/1	2/2	2/2	2/2	2/2	14 / 10	
	7	420	R	R	A	R	R	R	1	5
		997	A	A	A	A	R	A	5	1
		2470	A	A	A	A	A	A	6	0
		6000	A	A	A	A	A	A	6	0
	Accept/Reject		3/1	3/1	4/0	3/1	2/2	3/1	18 / 6	
	Accept/Reject (all months)		8/4	9/3	8/4	8/4	6/6	7/5	46 / 26	

*Accept (A) or Reject (R) hypothesis at the .05 level of significance for χ^2 variate with m degrees of freedom, $m = n-1-b$, where n = number of class intervals, b = number of parameters of the gamma distribution = 2.

SECTION V. CONCLUSIONS

This report has emphasized methods for establishing the validity of the hypothesis that observed gust variables, including gust component magnitude, gust length, u Range, and L Range, have been drawn from gamma distributed populations and that observed gust modulus has been drawn from a bivariate gamma distributed population that can be approximated with a Weibull distribution. An analytical procedure has been proposed for testing for the bivariate gamma distribution. The procedure has the advantage of not requiring frequency counts within narrow cells defined by the intersection of intervals of the marginal distribution; these frequency counts would be impractical and unreliable because of the limited sample size (150) of the available data. Instead, the new method requires theoretical and observed frequency counting over larger areas associated with non-dimensionalized and transformed variables. Preliminary results utilizing this method have indicated larger observed than expected frequencies for small gust lengths and associated small gust magnitudes; this is attributable to the definition of gust used in this study. These small gust lengths are not measurable with the Jimsphere system and as indicated in Section IV the results of hypothesis testing for the marginal distributions are improved greatly by eliminating them from the data sample. The hypothesis that gust component (u and v) magnitudes are drawn from a gamma distributed population is accepted at the .05 level of significance in 122 of the 136 cases tested; for gust length (Lu and Lv), 102 of the 136 cases were accepted.

The variables u Range and L Range have been used to represent component wind shear and shear interval associated with gusts. The hypothesis that u Range observations were drawn from a gamma distributed population was accepted at the .05 level in 66 of the 72 cases tested; the acceptance ratio was somewhat smaller for L Range with acceptance in 46 of the 72 cases tested.

Testing of the hypothesis that gust modulus is drawn from a Weibull distributed population has yielded highly favorable results with acceptance of the hypothesis at the .05 level in 65 of the 69 cases tested.

SECTION VI. REFERENCES

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2. Smith, O. E., and Adelfang, S. I.: A Model for Gust Amplitude and Gust Length Based on the Bivariate Gamma Probability Distribution Function. Presented at the AIAA 19th Aerospace Sciences Meeting, January 1981, St. Louis, Missouri. AIAA Paper 81-0299.
3. Justus, C. G., Hargraves, W. R., Mikhail, A., and Graber, D.: Methods for Estimating Wind Speed Frequency Distributions. JAM, Vol. 17, pp. 350-353, March 1978.

APPENDIX

Parameters γ and β (calculated from sample moments) for hypothetical gamma distributions of gust component variables $|u'|$, $|v'|$, Lu , Lv , u Range, and L Range defined in Section IV are listed in Tables A-1 through A-6.

The parameters in the tables can be used to derive the gamma probability density function of the form

$$g(x) = \frac{\beta^\gamma}{\Gamma(\gamma)} x^{\gamma-1} \text{EXP}(-\beta x) \quad (1)$$

Equation (1) can be expressed in terms of a nondimensional variable y , i.e., $y = \frac{x}{\beta}$, such that

$$g(y) = \frac{1}{\Gamma(\gamma)} y^{\gamma-1} \text{EXP}(-y) \quad (2)$$

The probability that y does not exceed a specified value, Y , is given by

$$P_r \{y \leq Y\} = \int_0^Y g(y) dy = \frac{1}{\Gamma(\gamma)} \int_0^Y y^{\gamma-1} \text{EXP}(-y) dy \quad (3)$$

The integral on the right side of Equation 3 is the incomplete gamma function, $H(\gamma, Y)$, which can be approximated with the series summation given by Equation 4 in Section II with the substitution

$$a = \gamma$$

$$X = Y$$

Table A-1. Gamma Distribution Parameters γ and β of Absolute u Component Gust
Estimated from Sample Moment Statistics*

Month	Filter λ_c (m)	Altitude (km)											
		4		6		8		10		12		14	
		γ	β (s/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	2.7977	7.1252	3.0595	8.2808	2.6387	6.5191	2.7884	6.1971	2.2586	4.3472	2.7237	4.4131
	997	3.6720	4.3885	3.0471	3.7889	2.7925	3.3954	2.6372	3.1212	2.5924	2.2480	2.7435	2.2276
	2470	3.4160	2.0864	3.2470	2.0025	3.3461	1.9497	3.0639	1.6141	2.3545	.8945	3.3385	1.1593
	6000	1.3784	.7212	2.5834	.9603	2.9254	.9140	2.4424	.6500	2.6651	.5478	4.1791	.8345
April	420	2.2160	6.8163	2.6129	7.8253	2.4453	7.7683	2.8283	8.4910	2.7139	6.0048	3.3506	5.3996
	997	2.8800	3.7674	3.9797	5.0300	2.9474	4.2228	2.9914	4.5243	3.0542	3.0214	2.8570	2.0031
	2470	3.2557	2.1546	3.3992	2.1361	3.5606	2.2367	3.1450	2.1659	3.2043	1.3532	3.1588	.9373
	6000	1.4722	1.1660	3.4500	1.2714	2.9691	1.1169	3.1542	1.0743	3.4673	.7980	4.3829	.7665
July	420	3.0155	9.7748	3.1550	9.3360	3.3174	10.4939	3.1022	10.6578	2.4241	7.9454	2.1704	4.7490
	997	3.0537	4.7798	2.9116	4.3739	3.9496	5.7012	3.0069	4.9926	3.2366	4.9563	3.0064	2.7846
	2470	3.0713	2.6264	4.0635	3.3023	3.2331	2.4762	2.6744	2.0260	2.7080	1.9894	3.0995	1.3168
	6000	2.3696	1.5587	3.6039	1.8240	2.9507	1.4285	2.6261	1.1393	2.8570	.8762	3.0678	.7489

* $\hat{\gamma} = (\bar{x}/\sigma)^2$

$\hat{\beta} = \hat{\gamma}/\bar{x}$

Table A-2. Gamma Distribution Parameters γ and β of Gust Length, Lu,
Estimated from Sample Moment Statistics*

Month	Filter λ_c (m)	Altitude (km)											
		4		6		8		10		12		14	
		γ	β (1/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	4.3144	.0303	4.7477	.0317	3.5532	.0262	3.9387	.0338	2.7808	.0259	2.3970	.0227
	997	5.0961	.0191	4.5724	.0173	4.3199	.0168	3.2904	.0143	3.4236	.0143	2.8273	.0120
	2470	3.5379	.0059	3.2987	.0057	2.8203	.0047	3.0325	.0047	2.9236	.0051	2.9723	.0046
	6000	2.2881	.0037	2.1950	.0020	2.0270	.0017	2.8136	.0020	1.9954	.0015	2.2030	.0017
April	420	3.8465	.0287	4.3296	.0320	4.4428	.0332	4.2881	.0337	3.6501	.0299	3.6396	.0265
	997	4.9658	.0190	5.6832	.0196	4.7895	.0172	3.9201	.0158	3.6556	.0146	4.4410	.0161
	2470	3.5516	.0059	3.0867	.0051	3.7403	.0059	2.4735	.0040	3.7749	.0059	3.7691	.0059
	6000	2.0203	.0020	2.9714	.0026	2.6921	.0023	2.9223	.0022	2.6511	.0018	2.4178	.0019
July	420	5.2320	.0367	6.1092	.0415	5.1708	.0379	4.1033	.0302	4.0764	.0321	3.6856	.0259
	997	3.9741	.0160	3.7563	.0146	5.1326	.0191	3.7614	.0139	4.4005	.0154	4.2432	.0145
	2470	3.0248	.0056	2.6623	.0048	2.9303	.0051	2.9148	.0047	2.8370	.0044	3.2452	.0050
	6000	1.9617	.0029	2.3853	.0023	2.1322	.0021	2.4274	.0022	3.6950	.0023	2.5452	.0019

* $\hat{\gamma} = (\bar{x}/\sigma)^2$

$\hat{\beta} = \bar{\gamma}/\bar{x}$

Table A-3. Gamma Distribution Parameters γ and β of Absolute v Component Gust
Estimated from Sample Moment Statistics*

Month	Filter λ_c (m)	Altitude (km)											
		4		6		8		10		12		14	
		γ	β (s/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	2.5620	6.3677	3.0842	8.7604	2.8059	7.5590	2.3220	5.0906	2.4888	3.3212	2.7977	3.2771
	997	3.1964	3.4383	3.3494	4.0559	3.3089	4.1330	2.3731	2.6057	3.1270	2.2114	3.1354	1.9026
	2470	3.4847	1.9691	3.6922	1.9896	2.8574	1.5778	2.8406	1.2632	2.9095	.9491	3.1772	.8909
	6000	2.2621	1.1301	3.0577	.9520	2.5705	.7725	2.0864	.4907	2.6933	.5152	3.1304	.5772
April	420	2.6285	7.1176	3.3563	8.2955	2.2458	6.4842	3.1248	9.3733	2.1858	3.8315	2.3588	3.0760
	997	3.3134	3.8589	4.8051	5.2364	2.8348	3.7631	3.4730	4.6320	2.3209	1.9349	2.8295	1.7494
	2470	3.5359	1.9731	3.4691	1.8528	3.0296	1.7655	2.7126	1.4948	2.9190	1.0677	3.1823	.8619
	6000	3.3614	1.7161	3.5757	1.1955	3.7780	1.2287	3.3991	.9762	2.5541	.5941	3.3257	.5816
July	420	3.4726	11.0219	3.6591	11.1260	3.2813	10.1280	2.5372	9.5756	2.9943	10.0276	2.2201	4.7901
	997	3.2128	5.3395	3.4127	5.0117	4.6553	6.7185	2.8180	4.5618	3.6180	5.3203	2.7254	2.3321
	2470	2.6506	2.1572	3.6841	2.9425	3.7912	2.9586	2.7195	2.0371	2.3367	1.5839	2.7649	1.0942
	6000	1.7155	1.2577	3.9569	1.9228	3.5513	1.7642	3.2635	1.4297	3.3935	1.1520	3.5878	.8447

* $\hat{\gamma} = (\bar{x}/\sigma)^2$

$\hat{\beta} = \hat{\gamma}/\bar{x}$

Table A-4. Gamma Distribution Parameters γ and β of Gust Length, Lv,
Estimated from Sample Moment Statistics*

Month	Filter λ_c (m)	Altitude (km)											
		4		6		8		10		12		14	
		γ	$\beta(1/m)$	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	4.2668	.0276	4.3845	.0329	4.6915	.0365	3.3980	.0300	2.8752	.0257	3.1804	.0327
	997	5.4600	.0189	5.2302	.0181	4.0927	.0153	2.7340	.0120	3.1941	.0134	2.5454	.0114
	2470	3.2958	.0057	4.2853	.0064	3.2358	.0052	3.1330	.0046	2.3618	.0039	2.8339	.0048
	6000	.8325	.0014	3.4616	.0029	2.8211	.0023	2.8833	.0023	2.5287	.0022	2.1470	.0021
April	420	4.4899	.0306	4.5662	.0283	4.0750	.0292	3.2733	.0282	2.7620	.0221	2.3890	.0212
	997	4.8218	.0166	6.8564	.0237	4.1517	.0146	3.7680	.0141	3.6093	.0150	2.8605	.0122
	2470	3.5608	.0056	3.7188	.0061	3.4439	.0050	3.4641	.0055	3.0020	.0049	3.0412	.0055
	6000	1.7891	.0027	3.4475	.0034	4.0519	.0034	3.0236	.0022	1.8364	.0017	3.4754	.0039
July	420	5.4864	.0401	5.5545	.0383	4.2290	.0288	4.2895	.0307	4.3395	.0306	3.7989	.0266
	997	4.6205	.0181	4.8734	.0183	5.7390	.0209	4.5953	.0164	4.9991	.0178	4.6217	.0158
	2470	3.0105	.0055	3.1497	.0057	3.4228	.0063	3.5367	.0057	2.7405	.0042	3.0952	.0050
	6000	1.2473	.0020	2.8210	.0028	2.4524	.0026	3.6418	.0031	3.6045	.0026	2.6019	.0022

* $\hat{\gamma} = (\bar{x}/\sigma)^2$

$\hat{\beta} = \bar{\gamma}/\bar{x}$

Table A-5. Gamma Distribution Parameters γ and β of u Range Estimated from Sample Moment Statistics*

Month	Filter λ_c (m)	Altitude (km)											
		4		6		8		10		12		14	
		γ	β (s/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	3.4988	4.7377	3.3888	4.6909	3.4400	4.3525	3.1709	3.6078	3.4380	3.3728	3.9035	3.4181
	997	4.0685	2.5484	3.1495	2.0366	3.2842	2.1506	2.8046	1.7967	2.6382	1.2044	2.9463	1.3421
	2470	3.8216	1.3669	3.1250	1.0598	3.2096	1.0171	2.6697	.83208	2.2354	.48073	3.0640	.5888
	6000	1.6391	.48268	2.5218	.52372	2.0067	.37436	2.0257	.30665	2.5443	.33335	3.5214	.4435
April	420	2.7635	4.4602	3.2748	5.3127	2.7241	4.6830	3.3368	5.4695	3.1181	3.5458	3.6009	3.0546
	997	3.3845	2.4067	4.0804	2.7498	3.7056	2.9160	3.3044	2.7948	2.7672	1.4059	3.5335	1.3483
	2470	3.3141	1.1613	3.3726	1.1900	3.7203	1.2763	2.6586	1.0445	2.5676	.57766	3.3004	.5385
	6000	2.7461	.73656	3.3468	.74479	3.2969	.69900	2.4936	.46891	3.4774	.45412	5.2357	.5934
July	420	3.5465	5.8122	3.7867	5.7765	3.8386	6.3667	3.1673	5.8122	2.6331	4.6180	2.4736	2.7753
	997	3.3767	2.8885	3.7713	2.9707	3.7837	2.9990	3.3852	3.0288	3.0171	2.4227	3.3480	1.5222
	2470	2.7029	1.2414	3.7104	1.7019	3.0316	1.3083	2.9147	1.2418	2.8121	1.1409	3.3170	.7530
	6000	2.4536	1.0152	3.2221	.97725	3.0657	.84117	2.3522	.57890	2.7642	.48606	2.4447	.3796

* $\hat{\gamma} = (\bar{x}/\sigma)^2$

$\hat{\beta} = \hat{\gamma}/\bar{x}$

Table A-6. Gamma Distribution Parameters γ and β of L Range Estimated from Sample Moment Statistics*

Month	Filter λ_c (m)	Altitude (km)											
		4		6		8		10		12		14	
		γ	β (1/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	3.5129	.0269	3.8620	.0318	3.4447	.0301	3.7603	.0376	2.1597	.0215	2.2610	.0240
	997	3.5618	.0147	3.2469	.0147	2.8123	.0124	2.2186	.0111	2.4612	.0117	1.9530	.0099
	2470	2.5643	.0053	2.2883	.0046	2.2325	.0042	1.7932	.0037	1.9100	.0042	2.3743	.0045
	6000	1.6415	.0030	1.5704	.0017	1.5307	.0016	1.5881	.0015	1.8765	.0022	2.0555	.0022
April	420	3.5102	.0293	3.9302	.0319	3.3691	.0288	3.2901	.0323	3.1661	.0293	2.2937	.0198
	997	3.2268	.0130	4.1280	.0168	2.6893	.0111	2.3915	.0116	2.5769	.0110	2.5890	.0108
	2470	2.7068	.0051	2.6437	.0051	1.1583	.0058	2.3049	.0048	2.9662	.0063	2.7228	.0051
	6000	1.6792	.0025	2.0037	.0021	1.9754	.0019	2.1335	.0021	2.1085	.0022	3.1420	.0039
July	420	5.1746	.0398	3.9245	.0287	3.8306	.0304	3.7633	.0320	3.6531	.0325	3.0540	.0240
	997	2.7890	.0121	2.8991	.0123	3.6725	.0154	2.4720	.0101	3.1053	.0128	3.1303	.0114
	2470	2.3830	.0053	2.0355	.0046	2.5076	.0054	2.2080	.0042	2.1111	.0040	2.6624	.0049
	6000	.9255	.0016	1.6771	.0022	2.0761	.0026	2.0056	.0020	2.8401	.0025	2.1455	.0026

* $\hat{\gamma} = (\bar{x}/\sigma)^2$

$\hat{\beta} = \hat{\gamma}/\bar{x}$